Reference sheet

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B \qquad \sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Vector calculus theorems

Suppose $\boldsymbol{F}$ is a C^1 vector field on $\mathbb{R}^3$.

Stokes' Theorem	$\oint_{\partial\Sigma} \boldsymbol{F}\cdot d\boldsymbol{r} = \iint_{\Sigma} (\nabla\times\boldsymbol{F})\cdot d\boldsymbol{S}$	for all surfaces $\Sigma \subset \mathbb{R}^3$
Divergence Theorem	$\oiint_{\partial\Omega} \boldsymbol{F}\cdot d\boldsymbol{S} = \iiint_{\Omega} (\nabla\cdot\boldsymbol{F})\, dV$	for all regions $\Omega \subset \mathbb{R}^3$

Vector calculus identities

$$\nabla\times(f\boldsymbol{F}) = (\nabla f)\times\boldsymbol{F} + f(\nabla\times\boldsymbol{F})$$
$$\nabla\cdot(f\boldsymbol{F}) = (\nabla f)\cdot\boldsymbol{F} + f(\nabla\cdot\boldsymbol{F})$$
$$\nabla\times(\nabla f) = \boldsymbol{0}$$
$$\nabla\cdot(\nabla\times\boldsymbol{F}) = 0$$
$$\nabla\times(\nabla\times\boldsymbol{F}) = \nabla(\nabla\cdot\boldsymbol{F}) - \nabla^2\boldsymbol{F}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$.

Maxwell's equations

$$\nabla\cdot\boldsymbol{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$$
$$\nabla\cdot\boldsymbol{B} = 0$$
$$\nabla\times\boldsymbol{B} = \mu_0\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t} + \mu_0\boldsymbol{J}$$

Complex trigonometric and hyperbolic identities For all complex numbers $z \in \mathbb{C}$:

$$\cos z = \frac{e^{jz} + e^{-jz}}{2} \qquad \sin z = \frac{e^{jz} - e^{-jz}}{2j} \qquad \cosh z = \frac{e^{z} + e^{-z}}{2} \qquad \sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\cos jz = \cosh z \qquad \sin jz = j \sinh z$$

Cauchy-Riemann equations .
In Cartesian form

$$u_x = v_y \qquad u_y = -v_x$$

and if f is differentiable the derivative is given by $f'(x + jy) = u_x(x, y) + jv_x(x, y)$.
In polar form

$$u_r = \frac{1}{r} v_\theta \qquad v_r = -\frac{1}{r} u_\theta \quad .$$

and if f is differentiable the derivative is given by $f'(re^{j\theta}) = e^{-j\theta}\big(u_r(r, \theta) + jv_r(r, \theta)\big)$.

Generalized Cauchy integral formula Suppose f is analytic everywhere in a simply connected region D. For any integer $n \geq 0$,

$$\oint_\Gamma \frac{f(z)}{(z - z_0)^{n+1}}\, dz = \frac{2\pi j}{n!} f^{(n)}(z_0)$$

where Γ is any simple closed contour going counterclockwise around the point z_0 in D.

Useful Taylor series

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$