MATH 271 WINTER 2015
MIDTERM

1. Use the Euclidean algorithm to find $\gcd(85, 58)$. Then use your work to write $\gcd(85, 58)$ in the form $85a + 58b$ where a and b are integers.

2. Prove or disprove the following statements.

(a) For all real numbers x and y, $\lceil xy \rceil = \lceil x \rceil \lfloor y \rfloor$.

(b) There exist real numbers x and y so that $\lceil xy \rceil = \lceil x \rceil \lfloor y \rfloor$.

3. Let $\mathcal{P}$ be the statement: "For all integers a and b, if $a \mid b^2$ then $a \mid b$."

(a) Is $\mathcal{P}$ true? Prove your answer.

(b) Write out the converse of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true? Prove your answer.

(c) Write the contrapositive of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.

4. Prove the statement "For all non-negative real numbers x, if x is irrational then $2\sqrt{x}$ is irrational" by contradiction.

5. Let A and B be sets so that $1 \notin A - B$ and $(1, 2) \in A \times B$.

(a) Which elements must be in A? Explain.

(b) Which elements must be in B? Explain.

(c) What is the smallest possible number of elements in $B - A$? Explain.

5. For each of the following, give a brief explanation on how you get the answers and simplify your answers to a number.

(a) In how many ways can two distinct integers be selected from the set $\{1, 2, 3, \ldots, 100\}$?

(b) In how many ways can two distinct integers be selected from the set $\{1, 2, 3, \ldots, 100\}$ so that their product is even?

7. Prove by induction that $5^n - 4n - 1$ is divisible by 16 for all integers $n \geq 1$.