

**MATH 271 WINTER 2016**  
**MIDTERM**

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1. Use the Euclidean algorithm to find  $\gcd(271, 98)$ . Then use your work to write  $\gcd(271, 98)$  in the form  $271a + 98b$  where  $a$  and  $b$  are integers.
2. Prove the statement: “For all real numbers  $x$ , if  $x$  is irrational then  $271x$  is irrational” by contradiction.
3. Let  $\mathcal{P}$  be the statement: “For all sets  $A$ ,  $B$ , and  $C$ , if  $A \cap B \subseteq C$  then  $A - C = \emptyset$ .”
  - (a) Is  $\mathcal{P}$  true? Prove your answer.
  - (b) Write out the converse of  $\mathcal{P}$ . Is the converse of  $\mathcal{P}$  true? Prove your answer.
  - (c) Write the contrapositive of  $\mathcal{P}$ . Is the contrapositive of  $\mathcal{P}$  true? Explain.
4. Let  $S = \{1, 2, 3, \dots, 10\}$  and  $T = \{1, 2, 3, \dots, 20\}$ .
  - (a) Is there a subset  $X$  of  $T$  so that  $4 \in X$  and  $X$  has exactly 4 elements? Explain.
  - (b) Is there a subset  $Y$  of  $T$  so that  $4 \in Y$ , and both  $Y - S$  and  $S - Y$  have exactly 4 elements? Explain.
  - (c) How many subsets  $X$  of  $T$  are there so that  $4 \in X$  and  $X$  has exactly 4 elements?
  - (d) How many subsets  $Y$  of  $T$  are there so that  $4 \in Y$ , and both  $Y - S$  and  $S - Y$  have exactly 4 elements?
5. Prove by induction that  $5^n - 4n - 1$  is divisible by 16 for all integers  $n \geq 1$ .