MATH 271 – Summer 2016
Assignment 1
Due at 10:00 on Tuesday, July 19.

Hand in your assignment to your TA by the beginning of the lab on the due date. Your solutions must be understandable to the marker (i.e., logically correct as well as legible), and must be done in your own words. Marked assignments will be handed back during your scheduled lab. Please make sure that:

- Your solutions have a cover page containing **only** your ID number and your TA's name,
- your name and ID number are on the top right corners of **all** the remaining pages, and
- the pages of your assignment are **stapled**.

Note: Answer all three problems, but only one problem per assignment will be marked for credit.

Problem 1.

You are wandering in a fictional forest that is inhabited by trolls. Each troll is either a knight or a knave. Knights always tell the truth and knaves always speak falsehoods. For each of the following situations, determine which trolls are knights and which are knaves. Write a proof using complete sentences that confirms your assertion.

(a) You encounter two trolls (X and Y) who make the following statements.

X: "Both of us are knaves."
Y: "Exactly one of us is a knave."

(b) You encounter three trolls (Q, R, and S) who make the following statements.

Q: "Exactly one of us is a knight."
R: "Exactly one of us is a knave."
S: "We are all knaves."

(c) You encounter three trolls (A, B, and C) who make the following statements.

A: "If I am a knave, then exactly two of us are knights."
B: "Troll A is a knave."
C: "At least one of us is a knight."

Problem 2.

For each true statement, give a proof. For each false statement, write the negation and prove that.

(a) $\forall x, y \in \mathbb{R}$, if $\lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor$ then $x \in \mathbb{Z}$ or $y \in \mathbb{Z}$.

(b) $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ so that $\lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor$.

(c) There exists a real number x so that x is not an integer, $x > 2016$, and $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$

(d) $\forall N \in \mathbb{Z}^+$, $\exists x \in \mathbb{R}$ so that $x \notin \mathbb{Z}$, $x > N$, and $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$

Problem 3.

Prove the following statements.

(a) $\forall a, b, d \in \mathbb{Z}$, if $d \mid a$ and $d \mid b$ then $d \mid (3a + 2b)$ and $d \mid (2a + b)$.

(b) $\forall a, b, d \in \mathbb{Z}$, if $d \mid (3a + 2b)$ and $d \mid (2a + b)$ then $d \mid a$ and $d \mid b$.

(c) $\forall a, b \in \mathbb{Z}^+$, $\gcd(a, b) = \gcd(3a + 2b, 2a + b)$.