MATH 271 – Summer 2016
Assignment 2
Due at 10:00 on Tuesday, July 26.

Hand in your assignment to your TA by the beginning of the lab on the due date. Your solutions must be understandable to the marker (i.e., logically correct as well as legible), and must be done in your own words. Marked assignments will be handed back during your scheduled lab. Please make sure that:

- your solutions have a cover page containing **only** your ID number and your TA's name,
- your name and ID number are on the top right corners of **all** the remaining pages, and
- the pages of your assignment are **stapled**.

Note: Answer all three problems, but only one problem per assignment will be marked for credit.

Problem 1.

Let N be your University of Calgary ID number.

(a) Use the Euclidean Algorithm to compute $\gcd(N, 271)$ and use this to find integers x and y so that $\gcd(N, 271) = Nx + 271y$.

(b) Suppose that M is an integer such that $\gcd(M, 271) = \gcd(M, 2016)$. Find $\gcd(M, 271)$. Explain how you get the answer.

(c) Suppose that K is an integer between $800,000$ and $900,000$ so that $\gcd(K, 271) > \gcd(K, 2016) > 250$. Find all possible values of K. Explain how you get the answers.

Problem 2.

Consider the sequence of Fibonacci numbers $f_1, f_2, f_3, \ldots$ which are defined as follows: $f_1 = 1$, $f_2 = 1$, and

$$f_n = f_{n-1} + f_{n-2}$$

for all integers $n \geq 3$.

(a) Prove that for all integers $n \geq 3$, $\gcd(f_n, f_{n-1}) = \gcd(f_{n-1}, f_{n-2})$. (You may use Lemma 4.8.2. No induction is needed.)

(b) Prove by weak induction that $\gcd(f_n, f_{n-1}) = 1$ for all integers $n \geq 2$. (Use part (a).)

(c) Prove by weak induction that $\sum_{i=1}^{n} (f_i)^2 = f_{n+1} f_n$ for all integers $n \geq 1$.

Problem 3.

The sequence $s_0, s_1, s_2, \ldots,$ is defined by $s_0 = 1$, and for all integers $n > 0$,

$$s_n = s_{\lfloor \frac{n}{2} \rfloor} + s_{\lfloor \frac{2n}{3} \rfloor} + n.$$

The sequence $t_0, t_1, t_2, \ldots,$ is defined by $t_0 = 2$, $t_1 = 3$, and for all integers $n > 0$, $t_n = 3t_{n-1} - 2t_{n-2}$.

(a) Find s_1, s_2, s_3, s_4, s_5, s_6, s_7, and s_8. Guess the smallest integer a so that $s_n > 4n$ for all integers $n \geq a$.

(b) Prove by strong induction that $s_n > 4n$ for all integers $n \geq a$, where a is the integer you chose in part (a).

(c) Find t_2, t_3, t_4, t_5, and t_6. Guess a formula for t_n.

(d) Prove by strong induction that your guess in part (c) is correct for all integers $n \geq 0$.