MATH 271 – Summer 2016
Assignment 3
Due at 10:00 on Tuesday, August 9.

Hand in your assignment to your TA by the beginning of the lab on the due date. Your solutions must be understandable to the marker (i.e., logically correct as well as legible), and must be done in your own words. Marked assignments will be handed back during your scheduled lab. Please make sure that:

- your solutions have a cover page containing **only** your ID number and your TA's name,
- your name and ID number are on the top right corners of **all** the remaining pages, and
- the pages of your assignment are **stapled**.

Note: Answer all three problems, but only one problem per assignment will be marked for credit.

Problem 1. For any sets X and Y, we define the *symmetric difference* of X and Y as

$$X \triangle Y = (X \cup Y) - (X \cap Y).$$

Note that it is also true that $X \triangle Y = (X - Y) \cup (Y - X)$. For each of the following statements, determine whether the statement is true or false. Prove the true statements using the element method. Prove the false statements false by giving a counterexample.

(a) "For all sets A, $A \triangle A = \emptyset$."

(b) "For all sets A, B, and C, if $A \subseteq B \cup C$ and $B \subseteq C \cup A$ then $A \triangle B = C$."

(c) "For all sets A, B, and C, if $A \triangle C = B \triangle C$ then $A = B$."

Hint: See the hint in the back of the textbook for problem 51 of section 6.3.

Problem 2. Consider the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. For parts (a) through (e), you must give a brief explanation on how you get the answer. Simplify your answer to a number.

(a) How many nonempty subsets of S have the property that the product of their elements is even? (For example, $T = \{1, 5, 8\}$ is a nonempty subset of S, and the product of the elements of T is $1 \cdot 5 \cdot 8 = 40$.)

(b) How many subsets of S have exactly 5 elements?

(c) How many subsets of S have 3 as their smallest element?

(d) How many subsets of S have 3 as their smallest element and have exactly 5 elements?

(e) How many subsets of S have 6 as their smallest element and have exactly 5 elements?

(f) Use the method of combinatorial proof to prove the following identity:

$$\binom{10}{5} = \binom{9}{4} + \binom{8}{4} + \binom{7}{4} + \binom{6}{4} + \binom{5}{4} + \binom{4}{4}.$$

Use complete sentences. (Hint: Find two different ways to count the number of subsets of S that have exactly 5 elements. Use parts (b), (d), and (e).)

Problem 3. Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = 2\lfloor x \rfloor - x$ for each $x \in \mathbb{R}$. Let $g\colon \mathbb{R} \to \mathbb{R}$ be the function defined by $g(x) = \frac{x}{x^2+1}$ for all $x \in \mathbb{R}$.

(a) Prove that f is one-to-one.

(b) Prove that f is onto.

(c) Is g one-to-one? Prove your answer.

(d) Is g onto? Prove your answer.