MATH 271 – Summer 2016
Assignment 4
Due at 10:00 on Tuesday, August 16.

Hand in your assignment to your TA by the beginning of the lab on the due date. Your solutions must be understandable to the marker (i.e., logically correct as well as legible), and must be done in your own words. Marked assignments will be handed back during your scheduled lab. Please make sure that:

- your solutions have a cover page containing **only** your ID number and your TA's name,
- your name and ID number are on the top right corners of **all** the remaining pages, and
- the pages of your assignment are **stapled**.

Note: Answer all three problems, but only one problem per assignment will be marked for credit.

Problem 1.

Let A, B, and C be some sets and suppose that $f\colon A \to B$ and $g\colon B \to C$ are functions. Prove or disprove each of the following statements.

(a) If f is one-to-one then $g \circ f$ is one-to-one.

(b) If both f and g are one-to-one then $g \circ f$ is one-to-one.

(c) If $g \circ f$ is one-to-one then f is one-to-one.

(d) If $g \circ f$ is one-to-one then g is one-to-one.

(e) If $g \circ f$ is one-to-one and f is onto then g is one-to-one.

Problem 2.

Consider the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let $\mathcal{F}$ denote the set of all functions from S to S. Define a relation R on $\mathcal{F}$ by:

$$\text{for all } f, g \in \mathcal{F}, \quad f\,R\,g \text{ if and only if } \exists x \in S \text{ so that } f(x) = g(x).$$

Let $\alpha \in \mathcal{F}$ be the function defined by $\alpha(x) = 1$ for each $x \in S$. Let $h \in \mathcal{F}$ be the function defined by $h(x) = \lfloor \frac{x+3}{2} \rfloor$ for each $x \in S$.

(a) Is R reflexive? Symmetric, Transitive? Prove your answers.

(b) Prove or disprove the statement: "$\exists f \in \mathcal{F}$ so that $\forall g \in \mathcal{F}$, $f\,R\,g$".

(c) How many functions $f \in \mathcal{F}$ are there so that $f\,R\,\alpha$? Explain.

(d) How many functions $f \in \mathcal{F}$ are there so that $f\,R\,h$? Explain.

(e) How many functions $f \in \mathcal{F}$ are there so that $f \not R\, \alpha$ or $f \not R\, h$? Explain.

Problem 3.

Consider the set $\mathbb{Z}^+$ of all positive integers. Let S be the relation on $\mathbb{Z}^+$ defined by

$$\text{for all } (a, b) \text{ and } (c, d) \text{ in } \mathbb{Z}^+ \times \mathbb{Z}^+ \qquad (a, b)\,S\,(c, d) \text{ if and only if } a + 2b = c + 2d$$

(a) Prove that S is an equivalence relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$.

(b) List all elements of $[(3, 3)]$ and all elements of $[(4, 4)]$.

(c) Is there an equivalence class of S that has exactly 271 elements? Explain.

(d) How many equivalence classes of S are there that contain at most 271 elements?