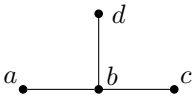
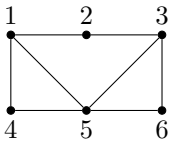
MATH 271 – Summer 2016
Extra practice problems – Week 6
University of Calgary
Mark Girard

Here are some questions to help you study graph theory for the final.

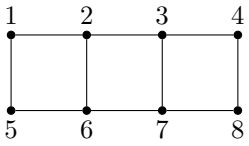
1. (a) Draw a simple graph with exactly 4 vertices and 6 edges.
   (b) Draw a simple graph with exactly 6 vertices and 4 edges and exactly two connected components.
   (c) Draw a simple graph with exactly 8 vertices, one of which has degree 6.
   (d) Does there exist a graph with exactly 8 vertices, so that three of the vertices have degree 3 and the remaining five vertices have degree 2? Explain.

2. Let G be the graph 

   (a) Is G bipartite?
   (b) Draw a **simple** graph H with exactly six vertices a, b, c, d, e, f and exactly seven edges and so that G is a subgraph of H.
   (c) Draw a **simple** graph F with exactly six vertices a, b, c, d, e, f so that G is a subgraph of F and F has an Euler circuit.

3. Consider the graph G given by: 

   (a) Is G bipartite?
   (b) Does G have an Euler trail?
   (c) Does G have an Euler circuit?
   (d) Does G have a Hamiltonian circuit?

4. Consider the graph given by: 

   (a) Is the graph bipartite?
   (b) Does this have an Euler trail?
   (c) Does this graph have an Euler circuit?
   (d) Does this graph have a Hamiltonian circuit?

5. (a) Draw a **simple** graph with exactly six vertices and exactly nine edges.
   (b) Draw a **simple** graph with exactly six vertices and exactly nine edges that is not bipartite but has an Euler circuit.
   (c) Draw a **simple** graph with exactly six vertices and exactly nine edges that is bipartite but does not have an Euler circuit.

6. (a) Draw a **simple** graph G with exactly seven vertices and exactly ten edges, and so that some vertex of G has degree 6.

   (b) Answer part (a) again, but so that your graph G does **not** have an Euler circuit. (Be sure to explain why you know that G does not have an Euler circuit.)

   (c) Answer part (a) again, but so that your graph G **does** have an Euler circuit. (Be sure to explain why you know that G has an Euler circuit.)

7. Let G be the graph with vertices labeled $\{1, 2, 3, 4, 5, 6\}$, and for any two vertices i and j, there is an edge connecting vertex i and vertex j if and only if $1 \leq |i - j| \leq 2$.

   (a) Draw the graph G.

   (b) Is G bipartite? Explain.

   (c) Does G have an Euler circuit? Explain.

   (d) Does G have an Euler trail? Explain.

   (e) Does G have a Hamiltonian circuit? Explain.

For more practice, try the following problems from the book which have solutions in the back.

- Section 10.1: Problems 1, 3, 8, 15, 17, 18, 21, 24, 37ab.
- Section 10.2: Problems 1, 4, 8a, 9a, 12, 14, 19, 23.