MATH 271 – Summer 2016
Study Guide for Final Exam
Topics list

- Proof techniques

  - How to prove different types of statements (i.e. $\forall$, $\exists$, 'if...then', 'or', 'and')
  - Negations, converses, contrapositives
  - Direct proof, proof by contradiction, proof by counterexample
  - Proof by cases

- Divisibility

  - Definition of divisibility
  - Definition of even and odd
  - How to prove facts about divisibility
  - Definition of prime and composite
  - Quotient-remainder theorem
  - common divisors and gcd

- Rational and irrational numbers

  - Definition of rational and irrational
  - How to prove facts about rational and irrational numbers

- Induction (regular, not strong)

  - How to use proof by induction (base case, induction step, IH, conclusion)
  - sequences and recursive sequences

- Sets

  - Definitions about sets (i.e. $\cup$, $\cap$, $\subseteq$, $=$, $\times$, $\mathcal{P}(\cdot)$, $-$, $|\cdot|$, $\emptyset$)
  - How to prove facts about sets (element method, counterexample)

- Counting

  - Multiplication rule and recipes for counting
  - Permutations, combinations
  - Inclusion/exclusion principles (i.e. $|A - B| = \ldots$ and $|A \cup B| = \ldots$)

- Functions

  - Definition of function
  - Definition (and how to prove) one-to-one and onto
  - Function composition: "$\circ$"

- Relations

  - Definition of relation
    * as a subset of $A \times A$
    * as a directed graph
  - Definitions of (and how to prove) when a relation is reflexive, symmetric, transitive
  - Equivalence relations
    * definition of and how to prove when a relation is an equivalence relation
    * equivalence classes and their elements

- Modular arithmetic
  - Definition of $a \equiv b \pmod{m}$
  - How to reduce a number modulo m
  - How to find inverses modulo m
  - Solve congruences modulo m (i.e. find x so that $3x + 4 \equiv 2 \pmod{m}$)
  - Euclidean algorithm to compute gcd

- Graphs
  - Definition of graph (vertex, edge, loop, adjacent, vertex degree, subgraph)
  - Definitions of simple, bipartite, and connected graphs
  - Definitions of walk, path, trail, circuit
  - Euler trails, Euler circuits, Hamiltonian trails, and Hamiltonian circuits

- Miscellaneous topics
  - Floor and ceiling functions ($\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$)
  - Types of numbers: $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$