**MATH 271 – Summer 2016**
Practice problems – Week 1

## Part I

For each of the following statements, determine whether the statement is true or false. Prove the true statements. For the false statement, write out its negation and prove that. For the conditional statements, write out the converse and the contrapositive. Determine whether they are true or false and give reasoning.

1. $\forall n \in \mathbb{Z}$, $n^2 + 2n$ is even.
2. $\exists n \in \mathbb{Z}$ such that $n^3 + n$ is odd.
3. $\forall x \in \mathbb{R}$, $x^2 - x \geq 0$.
4. $\forall n \in \mathbb{Z}$, $n^2 - n \geq 0$.
5. $\forall x, y \in \mathbb{Z}$, if $x^2 + 2x = y^2 + 2y$ then $x = y$.
6. $\forall x, y \in \mathbb{Z}$, if $2x^2 + x = 2y^2 + y$ then $x = y$.
7. $\forall a, b, c \in \mathbb{Z}$, if $a|(b+c)$ and $a|(b-c)$ then $a|b$ and $a|c$.
8. $\forall a, b, c \in \mathbb{Z}$, if $a|(b+c)$ and $a|(2b+c)$ then $a|b$ and $a|c$.
9. $\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ such that $n + m$ is even.
10. $\exists m \in \mathbb{Z}$ such that $\forall n \in \mathbb{Z}$, $n + m$ is even.
11. $\forall r \in \mathbb{Q}$, $\exists m \in \mathbb{Z}$ such that $rm \in \mathbb{Z}$.
12. $\exists m \in \mathbb{Z}$ such that $\forall r \in \mathbb{Q}$, $rm \in \mathbb{Z}$.
13. For all positive integers $n$, there exists a positive integer $m$ so that $3|(n+m)$.
14. There exists a positive integer $m$ so that for all positive integers $n$, $3|(n+m)$.

## Part II

For each of the following statements, prove or disprove the statement. Note that you can use the fact that $\sqrt{2}$ is irrational. For all other irrational numbers, you must prove that they are in fact irrational.

1. $\forall x, y \in \mathbb{R}$, $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.
2. $\exists$ a positive real number $a$ so that $\forall$ real numbers $x$, if $x - \lfloor x \rfloor < a$ then $\lfloor 3x \rfloor = 3\lfloor x \rfloor$.
3. $2 + \sqrt{2}$ is irrational.
4. $3\sqrt{2}$ is irrational.
5. $\forall x, y \in \mathbb{R}$, if $x$ and $y$ are irrational then $x + y$ is irrational.
6. $\forall x, y \in \mathbb{R}$, if $x$ and $y$ are irrational then $xy$ is irrational.
7. $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ so that $x + y$ is rational.
8. $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ so that $x + y$ is irrational.
9. $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ so that $xy$ is irrational.
10. $\forall x \in \mathbb{R}$ such that $x \neq 0$, $\exists y \in \mathbb{R}$ so that $xy$ is irrational.