MATH 271 – Summer 2016
Practice problems – Week 2

Part I

For each pair of integers a and b, use the Euclidean Algorithm to compute $\gcd(a, b)$ and find integers x and y such that $\gcd(a, b) = ax + by$.

1. $a = 156$ and $b = 115$.
2. $a = 132$ and $b = 76$.
3. $a = 2016$ and $b = 271$.

Part II

Use mathematical induction to prove the following statements.

1. For all integers $n \geq 1$, $\displaystyle\sum_{i=1}^{n}(i+1)2^i = n2^{n+1}$.
2. For all integers $n \geq 1$, $\displaystyle\sum_{i=1}^{n}\frac{1}{i(i+1)} = \frac{n}{n+1}$.
3. For all integers $n \geq 1$, the sum of the first n positive odd integers is equal to n^2.
4. For all integers $n \geq 0$, $3^n + 1$ is even.
5. For all integers $n \geq 1$, $5^n - 4n - 1$ is divisible by 16.
6. For all integers $n \geq 4$, $n! > 2^n$.
7. For all integers $n \geq 2$, $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}$.
8. For any real number $x > 1$ and all positive integers n, $(1+x)^n \geq 1 + nx$.

Part II

Use *strong* mathematical induction to prove the following statements.

1. The sequence $a_1, a_2, a_3, \ldots$ is defined by letting $a_1 = 3$, $a_2 = 5$ and $a_k = 3a_{k-1} - 2a_{k-2}$ for all integers $k \geq 3$. Prove that $a_n = 2^n + 1$ for all integers $n \geq 1$.
2. Consider the sequence defined by $t_1 = t_2 = t_3 = 1$ and $t_k = t_{k-1} + t_{k-2} + t_{k-3}$ for all $k \geq 4$. Prove that $t_n < 2^n$ for all $n \in \mathbb{Z}^+$.
3. Let a_n be the sequence defined by $a_1 = 1$, $a_2 = 8$, $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that $a_n = 3 \cdot 2^{n-1} + 2(-1)^n$ for all $n \in \mathbb{Z}^+$.
4. The sequence $s_1, s_2, s_3, \ldots$ is defined by: $s_1 = 1$, and for all integers $k \geq 2$, $s_k = 2 \cdot s_{\lfloor \frac{k}{2} \rfloor}$. Prove by induction that $s_n \leq n$ for all integers $n \geq 1$.