MATH 271 – Summer 2016

Practice problems – Week 3

Part I

Determine which of the following statements are true and which are false. Prove the true statements. For the false statements, write the negation and prove that. Use the element method for all proofs.

1. $\forall A \subseteq \mathbb{Z}$, $\exists B \subseteq \mathbb{Z}$ so that $1 \in B - A$.
2. $\forall A \subset \mathbb{Z}$, $\exists B \subseteq \mathbb{Z}$ so that $1 \notin B - A$.
3. For all sets A, B, and C, if $A \cup B = C$ then $C \in \mathcal{P}(A) \cup \mathcal{P}(B)$.
4. For all sets A, B, and C, if $A \cap B = C$ then $C \in \mathcal{P}(A) \cap \mathcal{P}(B)$.
5. For all sets A, B, and C, $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.
6. For all sets A, B, and C, $A \cup (B \cap C) \subseteq (A \cup B) \cap C$.
7. For all sets A, B, and C, if $A \times B = A \times C$ then $B = C$.
8. For all sets A, B, and C, if $A - B \subseteq C$ then $A - C \subseteq B$.
9. For all sets A, B, and C, if $A \cap B \subseteq C$ and $B \cap C \subseteq A$ then $C \cap A \subseteq B$.
10. For all sets A, B, and C, if $A - (B \cap C) = \emptyset$ then $A - C = \emptyset$.
11. For all sets A, B, and C, if $A - C = \emptyset$ then $A - (B \cap C) = \emptyset$.

Part II

1. Suppose A and B are arbitrary subsets of $\mathbb{Z}$ such that $(2, 3) \in A \times B$ and $(3, 4) \in A \times B$, but $(1, 3) \notin A \times B$.
   (a) Find another element in $A \times B$ that is not $(2, 3)$ or $(3, 4)$. Explain.
   (b) Find another element that is not in $A \times B$. Explain.
2. Suppose A and B are arbitrary subsets of $\mathbb{Z}$ such that $A \cap B = \{1\}$.
   (a) Find an element of $A \times B$. Explain why it is an element of $A \times B$.
   (b) Find an element of the complement $(A \times B)^c$. (Here, assume that the universal set is $\mathbb{Z} \times \mathbb{Z}$.) Explain.

Part III

1. Consider the set of 4-digit positive integers. How many of them...
   (a) ... are there total?
   (b) ... are odd?
   (c) ... have distinct digits?
   (d) ... are odd and have distinct digits?
   (e) ... are even and have distinct digits?
   (f) ... have their digits in strictly increasing order? (i.e. 1234)
   (g) ... have the property that the sum of their digits is even?
   (h) ... are odd and have the property that the sum of their digits is even?
   (i) ... are odd and don't have the property that the sum of their digits is even?