MATH 271 – Summer 2016
Quiz 1 – Solutions

1. (*4 points*) **Write the negation** (in good English) of each of the following statements. Answers of the form "It is not the case that . . . " are not acceptable.

(a) For all integers y, there exists an integer x so that $y = x^3 + x$.
Negation: There exists an integer y so that for all integers x, $y \neq x^3 + x$.

(b) For all integers x and y, if $\left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{y}{2} \right\rfloor$ then $x = y$.
Negation: There exists integers x and y so that $\left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{y}{2} \right\rfloor$ but $x \neq y$.

(c) There exists an integer n so that for all integers m, $2 \nmid (n - m)$ or $3 \nmid (n - m)$.
Negation: For all integers n, there exists an integer m so that $2 \mid (n-m)$ and $3 \mid (n-m)$.

(d) For all real numbers x and y, if x is rational and y is irrational then $x + y$ and xy are irrational.
Negation: There exist real numbers x and y so that x is rational and y is irrational but $x + y$ or xy is rational.

2. (*4 points*) Prove that the statement "There exists an integer n so that for all integers m, $n + m$ and $n + 2m$ are odd" is false by writing out the negation and proving that.
Negation: For all integers n, there exists an integer m so that $n + m$ or $n + 2m$ is even.
Proof (*of negation*). Let n be an arbitrary integer. Let $m = -n$. Then $n+m = n+(-n) = 0$, which is even. □

3. (*7 points*)
Let $\mathcal{P}$ be the statement "For all integers a, b, and c, if $a \mid (b + 2c)$ then $a \mid b$ and $a \mid c$."

(a) Prove that $\mathcal{P}$ is false.

Proof (*that $\mathcal{P}$ is false*). Let $a = 3$, $b = 1$, and $c = 1$. Then $b + 2c = 3$ and $3 \mid 3$ so $a \mid (b + 2c)$. But $3 \nmid 1$ so $a \nmid b$. □

(b) Write out the converse of $\mathcal{P}$. Prove that the converse of $\mathcal{P}$ is true.
Converse of $\mathcal{P}$: "For all integers a, b, and c, if $a \mid b$ and $a \mid c$ then $a \mid (b + 2c)$."
The converse is true.

Proof (*of converse*). Let a, b, and c be arbitrary integers. Suppose that $a \mid b$ and $a \mid c$. By definition of divisibility, there exist integers k and m so that $b = ka$ and $c = ma$. Now

$$\begin{aligned} b + 2c &= ka + 2(ma) \\ &= (k + 2m)a, \end{aligned}$$

where $k + 2m$ is an integer. Therefore $a \mid (b + 2c)$. □

(c) Write out the contrapositive of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.
Contrapositive of $\mathcal{P}$: "For all integers a, b, and c, if $a \nmid b$ or $a \nmid c$ then $a \nmid (b + 2c)$."
The contrapositive of $\mathcal{P}$ is false since it is logically equivalent to the original statement, which is false.