MATH 271 – Summer 2016
Quiz 1

Thursday, July 14. Duration: 50 minutes

1. (*4 points*) **Write the negation** (in good English) of each of the following statements. Answers of the form "It is not the case that ..." are not acceptable.

(a) For all integers y, there exists an integer x so that $y = x^3 + x$.

(b) For all integers x and y, if $\left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{y}{2} \right\rfloor$ then $x = y$.

(c) There exists an integer n so that for all integers m, $2 \nmid (n - m)$ or $3 \nmid (n - m)$.

(d) For all real numbers x and y, if x is rational and y is irrational then $x + y$ and xy are irrational.

2. (*4 points*) Prove that the statement "There exists an integer n so that for all integers m, $n + m$ and $n + 2m$ are odd" is false by writing out the negation and proving that.

Name:____________________________________ Id #:__________________________

3. (*7 points*)
Let $\mathcal{P}$ be the statement "For all integers a, b, and c, if $a \mid (b + 2c)$ then $a \mid b$ and $a \mid c$."

(a) Prove that $\mathcal{P}$ is false.

(b) Write out the converse of $\mathcal{P}$. Prove that the converse of $\mathcal{P}$ is true.

(c) Write out the contrapositive of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.