MATH 271 – Summer 2016
Midterm
August 5, 2016, at 10:00am
Time limit: 110 minutes
Instructor: Mark Girard

Name: ______________________________

ID number: ______________________________

TA: ______________________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page. Write your ID number on the top of every subsequent page, in case the pages become separated.

Show all of your work on each problem.

If you need more space, use the reverse side of the pages. Clearly indicate when you have done this.

You may **not** use your books, notes, calculator, or any other aids on this exam.

The use of personal electronic or communication devices is prohibited.

Do not write in the table below.

Problem	Points	Score
1	4	
2	10	
3	8	
4	10	
5	8	
6	10	
Total:	50	

1. (4 points) Use the Euclidean Algorithm to compute $\gcd(94, 49)$. Use this to find integers x and y so that $\gcd(94, 49) = 94x + 49y$.

2. (10 points) For this problem, use no facts about $|$ ("divides") other than its definition.
Let $\mathcal{S}$ be the statement: "For all integers a and b, if $a \mid b$ then $(6a) \mid (2b)$."

(a) Prove that $\mathcal{S}$ is false by writing out its negation and proving that.

(b) Write out the converse of $\mathcal{S}$. Prove that the converse is true.

(c) Write out the contrapositive of $\mathcal{S}$. Is the contrapositive true? Explain.

3. (8 points) Prove by induction that $\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$ for all integers $n \geq 1$.

4. (10 points) Of the following statements, **one is true and one is false**. Prove the true statement. For the false statement, write its negation and prove that. Use the element method.

   (a) For all sets A, B, and C, if $A \cap B = \emptyset$ and $C \subseteq B$ then $A \cap C = \emptyset$.

   (b) For all sets A, B, and C, $A - (B \cup C) = (A - B) \cup C$.

5. (8 points) In this problem, use no properties of rational and irrational numbers other than the definition of rational.

   (a) Use proof by contradiction to prove the following statement.
   "For all real numbers x, if x is irrational then $\frac{1}{x}$ is irrational."

   (b) Prove the following statement. You may use the result from part (a).
   "For all irrational numbers x, there exists an irrational number y so that xy is rational."

6. (10 points) Consider the following sets of integers:

$$A = \{x \in \mathbb{Z} \mid 10 \leq x \leq 100 \text{ and } x \text{ is even}\}$$

and

$$B = \{x \in \mathbb{Z} \mid 10 \leq x \leq 100 \text{ and the sum of the digits of } x \text{ is even}\}.$$

(Example: The sum of the digits of 12 is $1 + 2 = 3$.)
No explanation is needed for your answers for (a), (b), or (c).

(a) Give three elements of $A - B$.

(b) Give three elements of $B - A$.

(c) Give three elements of $\mathcal{P}(A \cap B)$.

(d) Give one element of $(A \times B) - (B \times A)$. Explain your answer.

(e) How many elements does $A - B$ have? Explain your answer.