ECE 206 Fall 2019
Practice Problems Week 3

1. Evaluate the following integrals.

   (a) $\iint_D y\,dA$, where D is the region bounded by the lines defined by the equations $y = 0$, $y = 1$, $y = x - 1$ and $y = -x - 1$.

   (b) $\iint_D (x^2 + y)\,dA$, where D is the triangle with vertices $(0, 0)$, $(2, 0)$, and $(2, 1)$.

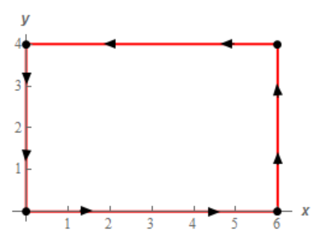
   (c) $\int_0^1 \int_{y^{1/3}}^1 \sqrt{1 + x^4}\,dx\,dy$

   (d) $\int_{-2}^0 \int_{-2}^x \frac{x}{\sqrt{x^2 + y^2}}\,dy\,dx$

2. Use Green's Theorem to compute the line integrals $\oint_\Gamma \boldsymbol{F} \cdot d\boldsymbol{r}$ by converting them into two-dimensional integrals for each of the following curves and fields.

   (a) $\boldsymbol{F}(x, y) = (\sqrt{1 + x^3}, 2xy)$ where Γ is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$, oriented counter clockwise.

   (b) $\boldsymbol{F}(x, y) = (y^4 - 2y, 6x + 4xy^3)$ where Γ is the curve indicated below.



3. Consider the region D that is bounded inside the curve Γ parameterized by $\boldsymbol{\gamma}(t) = \sin 2t\,\hat{\boldsymbol{\imath}} + \sin t\,\hat{\boldsymbol{\jmath}}$ for $0 \leq t \leq \pi$.

   (a) Sketch the curve and the region in the plane.

   (b) Use Green's Theorem to compute the area of D.

4. **Green's Theorem isn't always satisfied.**

   Consider the vector field $\boldsymbol{F}(x, y) = \frac{1}{x^2 + y^2}(-y, x)$ and let D be the region inside the unit circle.

   (a) Show that $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$, and thus that $\iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA = 0$.

   (b) Find the circulation of $\boldsymbol{F}$ around the unit circle ∂D using the definition.

   (c) What happened? Which is the true value of the circulation? (consider the assumptions of Green's theorem)