ECE 206 Fall 2019
Practice Problems Week 9

1. Simplify the following expressions using properties of the complex exponential function.
   (a) $e^{2\pm 3\pi j}$
   (b) $e^{z+\pi j}$ for arbitrary $z \in \mathbb{C}$.

2. Let f be a mapping of the complex plane and let $A \subseteq \mathbb{C}$ be a subset of $\mathbb{C}$ where f is defined. The *image* of A under f is the set of values $\{f(z) \mid z \in A\}$. For each set below, sketch the set then find and sketch its image under the given mapping.
   (a) the set $A = \left\{z \,\middle|\, \frac{5\pi}{3} < \mathrm{Im}(z) < \frac{8\pi}{3}\right\}$ under the mapping $f(z) = e^z$
   (b) the slit annulus $A = \left\{z \,\middle|\, \sqrt{e} \le |z| \le e^2 \text{ but } z \notin [-e^2, -\sqrt{e})\right\}$ under $f(z) = \mathrm{Log}(z)$

3. Solve the following for all possible values of z.
   (a) $e^z = -2$
   (b) $e^z = 1 + \sqrt{3}j$
   (c) $e^{2z-1} = 1$
   (d) $\sin z = 3j$
   (e) $\cos z = \cosh 4$
   (f) $|\tan z| = 1$
   (g) $\mathrm{Log}\, z = \frac{\pi}{2} j$

4. Find all possible values of the following. Then find the principal value of each.
   (a) $\log(-ej)$
   (b) $\log(1 - j)$
   (c) $\log e$
   (d) $(-1)^{1/\pi}$
   (e) $\left(\frac{e}{2}\left(-1 - \sqrt{3}j\right)\right)^{3\pi j}$

5. In class, we derived the formula $\sin^{-1}(z) = -j \log\left(jz + \sqrt{1 - z^2}\right)$ (where the equality is viewed as an equality of sets). Use similar methods to derive the following formulas.
   (a) $\cos^{-1}(z) = -j \log\left(z + \sqrt{z^2 - 1}\right)$
   (b) $\sinh^{-1}(z) = \log\left(z + \sqrt{1 + z^2}\right)$

6. Determine where the following mappings are differentiable, and find the derivative $f'(z)$ at those values.
   (a) $f(z) = z - \bar{z}$
   (b) $f(z) = x^2 + jy^2$
   (c) $f(z) = z\,\mathrm{Im}(z)$

7. Let $f : D \to \mathbb{C}$ be a complex-valued function on a domain $D \subseteq \mathbb{C}$. Show that if $f'(z) = 0$ everywhere in D, then f must be constant throughout D (i.e., there is some $\alpha \in \mathbb{C}$ such that $f(z) = \alpha$ for all $z \in D$).