

ECE 206 Fall 2019  
Practice Problems Week 11

1. Let  $\Gamma$  be the contour defined by the path  $\gamma(\theta) = e^{j\theta}$  for  $-\pi \leq \theta \leq \pi$ . Evaluate the following integrals.

(a)  $\int_{\Gamma} \text{Log } z \, dz$

(b)  $\int_{\Gamma} z^3 \text{Log } z \, dz$

2. Let  $\Gamma$  denote the circle  $|z - z_0| = R$ , taken counterclockwise. Compute the following integrals using the path  $\gamma(\theta) = z_0 + Re^{j\theta}$  for  $\theta \in (-\pi, \pi)$ .

(a)  $\int_{\Gamma_0} \frac{1}{z - z_0} \, dz$

(b)  $\int_{\Gamma_0} (z - z_0)^{n-1} \, dz$ , where  $n \in \mathbb{Z}, n \neq 0$

(c)  $\int_{\Gamma_0} (z - z_0)^{a-1} \, dz$  where  $a \in \mathbb{R}$  is a constant with  $a \neq 0$ . Here, we take  $(z - z_0)^{a-1}$  to be the principal value.

3. Use anti-derivatives to evaluate the following integrals.

(a)  $\int_j^{j/2} e^{\pi z} \, dz$

(b)  $\int_0^{\pi+2j} \cos\left(\frac{z}{2}\right) \, dz$

(c)  $\int_j^{3j} (z - 2j)^3 \, dz$

4. Let  $\Gamma$  be the circle  $|z| = 1$ . For which of the following functions is  $\int_{\Gamma} f(z) \, dz = 0$ ?

(a)  $f(z) = z^3$

(b)  $f(z) = \frac{e^z}{z}$

(c)  $f(z) = \frac{e^z}{z-2}$

(d)  $f(z) = \text{Log}(2z - 3j)$

5. Let  $\Gamma_1$  be the circle  $|z| = 1$  and  $\Gamma_2$  be the circle  $|z| = 3$ , each oriented counter clockwise. For which of the following functions does the equality  $\int_{\Gamma_1} f(z) \, dz = \int_{\Gamma_2} f(z) \, dz$  hold?

(a)  $f(z) = \frac{1}{z - 4j}$

(b)  $f(z) = \frac{z}{z + 2}$

(c)  $f(z) = \text{Log } z$

6. Use the Cauchy Integral Formula to evaluate the following integrals, where  $\Gamma$  is the circle  $|z| = 2$ .

(a)  $\int_{\Gamma} \frac{e^z}{z - j\frac{\pi}{2}} dz$

(b)  $\int_{\Gamma} \frac{z^2 + 1}{z(z^2 + 9)} dz$

(c)  $\oint_{\Gamma} \frac{z}{2z + 1} dz$

7. Use the generalized Cauchy Integral Formula to evaluate the following, where  $\Gamma$  is the square with edges on  $x = \pm 2$  and  $y = \pm 2$ .

(a)  $\oint_{\Gamma} \frac{\tan(\frac{z}{2})}{(z - \frac{\pi}{2})^2} dz$

(b)  $\int_{\Gamma} \frac{ze^z}{(z - 1)^4} dz$

8. Let  $\Gamma$  be the circle  $|z - j| = 2$ . Evaluate

(a)  $\int_{\Gamma} \frac{1}{z^2 + 4} dz$

(b)  $\int_{\Gamma} \frac{1}{(z^2 + 4)^2} dz$

9. Let  $\Gamma$  be the unit circle parameterized by  $z = e^{j\theta}$  for  $-\pi \leq \theta \leq \pi$ . Show that for any real constant  $a$ ,

$$\int_{\Gamma} \frac{e^{az}}{z} dz = 2\pi j$$

10. Let  $\Gamma$  be the circle  $|z| = 2$ . Evaluate  $\int_{\Gamma} \frac{\sin z}{z^2 + 1} dz$ .