ECE 206 – Fall 2019

Quiz 1

September 18, 2018 at 17:30

Instructor: Mark Girard
University of Waterloo

Notes:

1. Fill in your name (first and last) and student ID number.
2. This quiz contains 5 pages (including this cover page) and 5 problems. Check to see if any pages are missing.
3. Answer all questions in the space provided. Extra space is provided on the last page. If you want the overflow page marked, be sure to clearly indicate that your solution continues.
4. Show all of your work on each problem.
5. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.**
6. You may **not** use your books, notes, calculator, or any other aids on this quiz. The use of personal electronic or communication devices is prohibited.

Problem	Points
1	4
2	4
3	4
4	5
5	8
Total:	25

[4] 1. Consider a particle that travels along the path $\boldsymbol{\gamma} : [0,3] \to \mathbb{R}^3$ defined as

$$\boldsymbol{\gamma}(t) = \hat{\boldsymbol{\imath}} + t\,\hat{\boldsymbol{\jmath}} + \frac{2}{3}t^{3/2}\,\hat{\boldsymbol{k}} \qquad \text{for all } t \in [0,3].$$

Determine the distance traveled by the particle over the interval $0 \leq t \leq 3$.

[4] 2. Two particles travel with paths $\boldsymbol{\gamma}_1(t) = (\cos t, \sin t, t)$ and $\boldsymbol{\gamma}_2(t) = (1+t, t^2, t^3)$ for $t \geq 0$. Show that they start out at the same point. What is the angle between their initial velocities?

[4] 3. Find an equation for the flow lines of the vector field $\boldsymbol{F} : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\boldsymbol{F}(x, y) = (y, -x)$. Make a sketch of the field and flow lines, clearly indicating direction.

[5] 4. Evaluate $\displaystyle\int_\Gamma \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}$ is the vector field $\boldsymbol{F}(x, y, z) = (5z^2, 2x, x+2y)$ and Γ is the oriented curve parameterized by

$$\gamma(t) = (t, t^2, t^2)$$

for $0 \le t \le 1$.

[8] 5. Let Γ be the curve that is the intersection of the elliptical cylinder $\dfrac{y^2}{4} + z^2 = 1$ with the plane $x + y = 2$, and is oriented in the counterclockwise direction as viewed from the positive x-axis. Evaluate $\displaystyle\oint_\Gamma \boldsymbol{F} \cdot d\boldsymbol{r}$, where $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $\boldsymbol{F}(x, y, z) = (xz + yz, z, 2y)$.

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

This space is for sketch work or overflow

(If you want something here marked, be sure to clearly indicate on the question page.)