ECE 206 – Fall 2019

Quiz 1

September 18, 2018 at 17:30

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Notes:

1. Fill in your name (first and last) and student ID number.
2. This quiz contains 5 pages (including this cover page) and 5 problems. Check to see if any pages are missing.
3. Answer all questions in the space provided. Extra space is provided on the last page. If you want the overflow page marked, be sure to clearly indicate that your solution continues.
4. Show all of your work on each problem.
5. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.**
6. You may **not** use your books, notes, calculator, or any other aids on this quiz. The use of personal electronic or communication devices is prohibited.

Problem	Points
1	4
2	5
3	4
4	5
5	7
Total:	25

[4] 1. Evaluate the double integral $\iint_D x^2\, dA$, where D is the region that is interior of the triangle whose vertices are $(0,0)$, $(1,1)$, and $(-1,1)$.

[5] 2. Evaluate the integral $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y}\, dy\, dx$ by changing the order of integration.

[4] 3. Compute the area inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's Theorem and the vector field $\boldsymbol{F}(x, y) = (-y, x)$.

[5] 4. Let $\boldsymbol{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be the vector field defined as $\boldsymbol{F}(x, y) = (y^3 - xy,\, 3xy^2 + 2)$ and let Γ be the closed curve consisting of the part of the parabola $y = 1 - x^2$ from $-1 \leq x \leq 1$ and the part of the x-axis from $-1 \leq x \leq 1$, oriented counterclockwise. Compute the circulation of $\boldsymbol{F}$ around Γ using Green's Theorem.

(Hint: Parametrization of the curve would be tedious. . .)

5. Let $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field defined by

$$\boldsymbol{F}(x, y, z) = (2xy + y^2)\hat{\boldsymbol{\imath}} + (x^2 + 2xy + z^2)\hat{\boldsymbol{\jmath}} + (2yz - 3)\,\hat{\boldsymbol{k}}.$$

[4] (a) Find a scalar potential function for $\boldsymbol{F}$.

[3] (b) Compute the line integral of $\boldsymbol{F}$ along the curve defined by the path $\boldsymbol{\gamma}(t) = \big(2\cos(\pi t),\ t - 1,\ t^4\big)$ for $t \in [0, 1]$. (Hint: The line integral is messy...)

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

This space is for sketch work or overflow

(If you want something here marked, be sure to clearly indicate on the question page.)