ECE 206 – Fall 2019

Quiz 3

October 9, 2018 at 17:30

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Notes:

1. Fill in your name (first and last) and student ID number.
2. This quiz contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing.
3. Answer all questions in the space provided. Extra space is provided on the last page. If you want the overflow page marked, be sure to clearly indicate that your solution continues.
4. Show all of your work on each problem.
5. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.**
6. You may **not** use your books, notes, calculator, or any other aids on this quiz. The use of personal electronic or communication devices is prohibited.

Problem	Points
1	8
2	4
3	5
4	6
Total:	23

1. Consider a surface $\Sigma = \{\mathbf{\Phi}(u,v) \mid u \in (0,1] \text{ and } v \in [-\frac{\pi}{2}, \frac{\pi}{2}]\}$ determined by a parameterization

$$\mathbf{\Phi}(u,v) = (2u\sin v,\ u\cos v,\ u+1)$$

for all $0 < u \leq 1$ and $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$.

[2] (a) Determine the normal vector $\boldsymbol{n}_{\mathbf{\Phi}}(u,v)$ to the surface Σ.

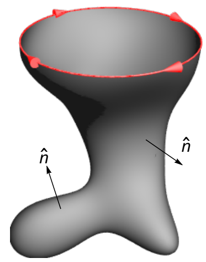
[3] (b) Determine an implicit representation for the tangent plane at the point $\boldsymbol{r}_0 = (0,1,2)$ on the surface Σ by describing the plane in terms of an equation of the form $\_x + \_y + \_z = \_$.

[3] (c) Describe the four grid curves of $\mathbf{\Phi}$ along $u = 1$, and along $v = -\frac{\pi}{2}$, $v = 0$, and $v = \frac{\pi}{2}$. (You may include a sketch as part of your answer.)

[4] 2. Evaluate the double integral $\iint_D y^2 \, dA$ where $D \subseteq \mathbb{R}^2$ is the region defined by $1 \leq x^2 + y^2 \leq 4$.

[5] 3. Let $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field defined by $\boldsymbol{F}(x, y, z) = (x, y, xyz)$ and let Σ be the surface given by $z = x^2 + y^2$ with $z \in [0, 4]$. Determine the flux of $\boldsymbol{F}$ downward through the surface Σ.

4. Consider the surface Σ (oriented outwards) shown below. The boundary of Σ is the circle $x^2 + y^2 = 1$ in the xy-plane (i.e., $z = 0$) oriented clockwise when viewed from above.



Σ

Let $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ and $\boldsymbol{G} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector fields defined by

$$\boldsymbol{F}(x, y, z) = (yz, xz + x, z), \qquad \text{and} \qquad \boldsymbol{G}(x, y, z) = (ye^{xy}, xe^{xy}, 0).$$

For each part, circle the best answer. Show your work to receive full credit. (Hint: Make use of an important theorem and find a simpler surface with the same boundary.)

[3] (a) Evaluate the integral $\displaystyle\iint_\Sigma (\nabla \times \boldsymbol{F}) \cdot \hat{\boldsymbol{n}}\, dA$.

-2π	$-\pi$	0	π	2π

[3] (b) Evaluate the integral $\displaystyle\oint_{\partial\Sigma} \boldsymbol{G} \cdot d\boldsymbol{r}$.

-2π	$-\pi$	0	π	2π

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B \qquad \sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Change of variable formula
If $\mathbf{\Phi}(u,v) = (x(u,v), y(u,v))$ is a one-to-one C^1-transformation $\mathbf{\Phi} : D \to \mathbb{R}^2$ then

$$\iint_{\mathbf{\Phi}(D)} f(x,y)\,dx\,dy = \iint_D f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

where the region $\mathbf{\Phi}(D) = \{\mathbf{\Phi}(u,v) \,|\, (u,v) \in D\}$ is the region mapped to. The Jacobian is defined as

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}.$$

Polar coordinates
For the coordinate transformation defined by $x = r\cos\theta$ and $y = r\sin\theta$, the Jacobian is

$$\frac{\partial(x,y)}{\partial(r,\theta)} = r.$$

Vector calculus identities

$$\nabla \times (f\boldsymbol{F}) = (\nabla f) \times \boldsymbol{F} + f(\nabla \times \boldsymbol{F})$$
$$\nabla \cdot (f\boldsymbol{F}) = (\nabla f) \cdot \boldsymbol{F} + f(\nabla \cdot \boldsymbol{F})$$
$$\nabla \times (\nabla f) = \mathbf{0}$$
$$\nabla \cdot (\nabla \times \boldsymbol{F}) = 0$$
$$\nabla \times (\nabla \times \boldsymbol{F}) = \nabla(\nabla \cdot \boldsymbol{F}) - \nabla^2\boldsymbol{F}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$.

This space is for sketch work or overflow

(If you want something here marked, be sure to clearly indicate on the question page.)