MATH 135 — Fall 2021
Practice Problems – Chapter 5

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Part I

Determine which of the following statements are true and which are false. Prove the true statements. For the false statements, write the negation and prove that.

1. $\forall A \subseteq \mathbb{Z}, \exists B \subseteq \mathbb{Z}$ so that $1 \in B - A$.
2. $\forall A \subset \mathbb{Z}, \exists B \subseteq \mathbb{Z}$ so that $1 \notin B - A$.
3. For all sets A, B, and C, $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.
4. For all sets A, B, and C, $A \cup (B \cap C) \subseteq (A \cup B) \cap C$.
5. For all sets A, B, and C, if $A \times B = A \times C$ then $B = C$.
6. For all sets A, B, and C, if $A - B \subseteq C$ then $A - C \subseteq B$.
7. For all sets A, B, and C, if $A \cap B \subseteq C$ and $B \cap C \subseteq A$ then $C \cap A \subseteq B$.
8. For all sets A, B, and C, if $A - (B \cap C) = \emptyset$ then $A - C = \emptyset$.
9. For all sets A, B, and C, if $A - C = \emptyset$ then $A - (B \cap C) = \emptyset$.

Part II

1. Proof De Morgan's Laws for sets. That is, for all sets A and B, it holds that:
   (a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$, and
   (b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
2. Suppose A and B are arbitrary subsets of $\mathbb{Z}$ such that $(2,3) \in A \times B$ and $(3,4) \in A \times B$, but that $(1,3) \notin A \times B$.
   (a) Find another element in $A \times B$ that is not $(2,3)$ or $(3,4)$. Explain.
   (b) Find another element that is not in $A \times B$. Explain.
3. Suppose A and B are arbitrary subsets of $\mathbb{Z}$ such that $A \cap B = \{1\}$.

(a) Find an element of $A \times B$. Explain why it is an element of $A \times B$.

(b) Find an element of the complement $\overline{A \times B}$. (Here, assume that the universal set is $\mathbb{Z} \times \mathbb{Z}$.) Explain.