MATH 137 — Fall 2020
Practice Problems

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1. Prove using only the definition of the limit that $\lim_{x\to 4}(x^2 - 3x + 2) = 6$.

2. Suppose $a_1, a_2, a_3 \ldots$ is a sequence that converges to 2, and suppose further that $a_n \neq 5$ for all $n \in \mathbb{N}$. Show (using only the definition of the limit) that $\lim_{n\to\infty} \dfrac{3}{5 - a_n} = 1$.

3. Let $L \in \mathbb{R}$ and let f be a function such that $\lim_{x\to\infty} f(x) = L$. For each of the following statements, either prove it is true (using only the definition of the limit) or show it is false by providing a counterexample.

   (a) $\lim_{x\to 0^+} f\left(\dfrac{1}{x}\right) = L$.

   (b) $\lim_{x\to 0} f\left(\dfrac{1}{x}\right) = L$.

4. Let f be a function such that $|f(x)| \leq x^2$ for all $x \in \mathbb{R}$. Prove that f is differentiable at $x = 0$ and that $f'(0) = 0$.

5. For each of the following functions, find all of the following: (i) the domain, (ii) the range, (iii) critical points, (iv) intervals where the function is increasing/decreasing, (v) intervals where the function is concave up/down, (vi) inflection points, (vii) local minima/maxima, (viii) global minimum/maximum (if they exist).

   (a) $f(x) = x - 3x^{1/3}$

   (b) $f(x) = \frac{3}{2-e^x}$

6. Consider the limit $\lim_{x\to\infty} \dfrac{x + \sin x}{x + 1}$.

   (a) Explain which conditions are satisfied for applying L'hopital's rule for this limit. What happens when you apply L'hopital's rule?

   (b) Use another method to compute this limit.

   (c) What conditions for applying L'hopital's rule are *NOT* met for this limit? Use this to explain why the answers from (a) and (b) are different.